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# Al-Hawārī's commentary on Ibn al-Bannā's *Talkhīṣ*: Contents and influences

Mahdi Abdeljaouad and Jeffrey A. Oaks

## **Abstract:**

In 1305 al-Hawārī completed his commentary on Ibn al-Bannā's famous arithmetic book *Talkhīṣ a'māl al-ḥisāb*. This is the only commentary, apart from Ibn al-Bannā's own, to have been written during the author's lifetime. What distinguishes al-Hawārī's book from the numerous later commentaries is its focus on numerical examples of the rules of calculation. We present here what we know about the author, his book, its salient features, and its influences.

**Keywords:** al-Hawārī, Ibn al-Bannā', Ibn Ghāzī, Şeker Zāde, arithmetic, double false position, algebra.

The famous mathematician and jurist Ibn al-Bannā' (1256-1321) wrote his *Talkhīṣ a'māl al-ḥisāb* (*Condensed [Book] on the Operations of Arithmetic*) as a concise, easy to memorize introduction to the rules for calculating with Indian numerals on whole numbers, fractions, and roots, and for finding unknown numbers by double false position and algebra. The work was completed sometime before 1301 and became one of the most popular textbooks in Western Islam. It has been widely accessible

to scholars since it was edited and translated into French first by Aristide Marre in 1865, then again by Mohammad Souissi in 1969.<sup>1</sup>

Popular and brief books naturally inspired commentaries. In fact, the first commentary of the *Talkhīṣ* was written by Ibn al-Bannā' himself. In 701H/1301 he completed his *Raf' al-ḥijāb 'an wujūh a'māl al-ḥisāb* (*Removal of the Veil from the Face of the Workings of Arithmetic*) in which he expanded the text of the *Talkhīṣ* by adding further explanations of the techniques, some numerical examples, and elements of number theory and combinatorics.<sup>2</sup> Among the many authors who composed commentaries of the *Talkhīṣ* after the death of the author are al-Ghurbī (2nd half 14th c.), al-Mawāhidī (ca. 1382), Ibn Zakariyyā (d. 1403-4), Ibn Qunfudh (d. 1407-8), al-'Uqbānī (d. 1408), Ibn Haydūr (d. 1413), Ibn Majdī (d. 1447), al-Ḥabbāk (d. 1463), al-Qalaṣādī (d. 1486), and Muḥammad al-Ghazzī (16th c.). In the other direction, Ibn al-Hā'im (d. ca. 1412) wrote an even more condensed version of the *Talkhīṣ*, and Ibn Marzūq (d. 1438), al-Wansharīsī (d. 1548-9), Ibn Ghāzī (d. 1513), and Ibn al-Qāḍī (d. 1616) put the contents of the *Talkhīṣ* to verse.<sup>3</sup>

One person is known to have written a commentary of the *Talkhīṣ* while the author was still living. Al-Hawārī (fl. early 14th c.), one of Ibn al-Bannā''s students, completed his *al-Lubāb fī sharḥ Talkhīṣ a'māl al-ḥisāb* (*The Essential Commentary on Condensed [Book] on the Operations of Arithmetic*) in 1305. We are currently preparing an edition, translation, and commentary of this book, and in this article we present what is known about the author, followed by salient features of the work and its later influences.

<sup>1</sup> Marre 1865; Ibn al-Bannā' 1969.

<sup>2</sup> Ibn al-Bannā' 1994; Aballagh & Djebbar 2001, 59, 99-104. Lamrabet 1994, 82 mistakenly attributes the date of AH 701 to the *Talkhīṣ*.

<sup>3</sup> See: al-Ghurbī L #399, al-Mawāhidī L #414, Ibn Qunfudh L #425 RI #780, Ibn Zakariyyā RI #793 [M1], al-'Uqbānī L #428 RI #781, Ibn al-Hā'im RI #783, Ibn Haydūr L #429, Ibn Marzūq L #435, Ibn Majdī RI #815, al-Ḥabbāk L #445 RI #831, al-Qalaṣādī L #454 RI #865, Ibn Ghāzī L #468 RI #913, al-Wansharīsī L #475, Ibn al-Qāḍī L #514, Muḥammad al-Ghazzī RI #998. References: "L #428" refers to mathematician #428 in Lamrabet 1994, and "RI #831" is mathematician #831 in Rosenfeld & Ĩhsanoğlu 2003.

## The author

Al-Hawārī's full name is given in the manuscripts of his book as 'Abd al-'Azīz ibn 'Alī ibn Dāwud al-Hawārī al-Miṣrātī.<sup>4</sup> The *kunya* "al-Miṣrātī" refers to the tribe or the city of the Libyan Miṣrāta. Following Lamrabet and Cheddadi, this implies that al-Hawārī hailed from the Berber tribe named Hawārī which emigrated from Libya to Morocco before the ninth century.<sup>5</sup>

Al-Hawārī relates in his chapter on double false position that Ibn al-Bannā dictated to him certain procedures "on Wednesday, the twenty-eighth of the month of Rajab in this year". This can only be the year 704H, so the date corresponds to Gregorian February 24, 1305, and it places al-Hawārī in Marrakesh at the time. Less than four months later he finished writing his commentary. The Madina MS reports that it was completed on Saturday, 18 Dhū al-Qa'da, 704H, which corresponds to Gregorian June 12, 1305.

Al-Hawārī is not known to have written any other books, and we know nothing more of his life or career.<sup>6</sup> Four modern sources give the date of his death as 744 or 745H, but we have not found any support for this in the manuscripts.<sup>7</sup>

<sup>4</sup> Some manuscripts show minor variations due to copy errors, such as "al-Hawāzī" for "al-Hawārī" and "Dāwūd" for "Dāwud". Also, some historians write "al-Huwārī". The only MS we have seen that indicates the vowel is the title page of Escorial 953/1, reproduced in [Ibn al-Bannā' 1969]. There it is "al-Hawārī". The vocalization "Hawārī" (as opposed to "Huwārī") is indicated on the title page of Madrid, Escorial MS 953/1.

<sup>5</sup> [Lamrabet 1994, 97-8; Ibn Khaldūn 2006 vol. 3, 460]. The Moroccan historian al-Mannouni shows the *kunya* instead as al-Miṣrātī, with an "s", not "ص" [Mannouni 1977, 28]. This orthography is used in [Ibn al-Bannā' 1969] and repeated by Lamrabet, and [Aballagh & Djebbar 2001]. All the MSS show it as "ص".

<sup>6</sup> Lamrabet indicates that al-Hawārī later settled in the city of Sebta (Ceuta) on the Strait of Gibraltar where he served as a civil notary and gave courses to some known contemporaries [Lamrabet 2008, 34]. Ibn al-Khaṭīb gives the full name of the notary as Abū Fāris 'Abd al-'Azīz al-Hawārī. Ibn al-Qāḍī gives another name: 'Abd al-'Azīz ibn Ibrāhīm ibn 'Abd al-'Azīz ibn Aḥmad Abū Fāris al-Hawārī al-Jazīrī al-Sibtī (1220-1301) [Ibn al-Qāḍī 1977 vol. 2, 133-4]. A well known notary ('*adl*') in Sebta, al-Hawārī taught many students around 689H (1290-1): Qāsim Ibn Yūsuf al-Tujībī (1271-1329), Shaykh Abū al-Ḥasan ibn Sulayman al-Qurṭubī (d. 1329) and Muḥammad Ibn Jābir (1274-1348). These two characters do not match our al-Hawārī since their father and grand-father differ.

<sup>7</sup> In their catalogs of manuscripts in Madina both Tashkandy and 'Umar Riḍā Kaḥḥāla give the year of al-Hawārī's death as 745H (1344-5). The Madina MS is dated the following

## The manuscripts

We consulted five manuscripts for our edition:

Madina, MS Ḥikmat 21 ḥisāb. 63 ff, 16 lines per page, 16 x 21 cm. This is followed by a short work of one page commencing “What was said about subtraction by seven”. The title is given in the explicit as: *al-Lubāb fī sharḥ Talkhīṣ a‘māl al-ḥisāb li Ibn al-Bannā’* (“The Essence of Commentary on Condensed [Book] on the Operations of Arithmetic by Ibn al-Bannā’”)<sup>8</sup>. This manuscript is the oldest of the four. The copyist completed it on 18 Rabī‘ I, 746H (Gregorian July 19, 1345).

Istanbul, Süleymaniye Library, MS Şehit Ali Paşa 1977/2, ff. 54a-103b. Copied 20 Ramaḍān 880H/11 January 1476 in Constantinople. This MS carries the title *Sharḥ al-Talkhīṣ fī al-ḥisāb lī al-Hawārī* (“Commentary on the Condensed [Book] on Arithmetic by al-Hawārī”). The copyist distinguished Ibn al-Bannā’s text from al-Hawārī’s comments by placing in front of Ibn al-Bannā’s extracts the letter “ص” (*ṣād*), which stands for *muṣannaf* (original text),<sup>9</sup> while passages of al-Hawārī are preceded by the letter “ش” (*shīn*), which is the first letter of the word *shārḥ* (commentary). The London, India Office MS (see below) employs the “ص” and “ش” similarly.<sup>10</sup>

Oxford, MS Marsh 378/3, ff. 109a-162a. Dated 1444 according to Woepcke.<sup>11</sup> Here the title is *Kitāb al-lubāb, wa huwa sharḥ al-Talkhīṣ li Ibn al-Bannā’ fī ‘ilm<sup>12</sup> al-ḥisāb* (“Essential Book, which is

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year, but there is nothing in it to indicate whether the author was still living or not. Lamrabet gives the date “vers 744H/1345”, and Bābānī writes “745” [Riḍā Kaḥḥāla 1973 vol. 4, 894; Tashkandy 1974, 110; Lamrabet 1994, 97; Bābānī 1951 vol. 1, 582].

<sup>8</sup> This is the same title of the Cairo manuscript (see below).

<sup>9</sup> [Ben Cheneb 1920].

<sup>10</sup> [Loth 1877, 225].

<sup>11</sup> [Ibn al-Bannā’ 1969, 8].

<sup>12</sup> MS has “*al-‘ilm*” in error.

a Commentary on the Condensed [Book] of Ibn al-Bannā' on the Science of Arithmetic").

Tunis, National Library of Tunis MS 9940. 32 ff., 22 x 26 cm, 29 lines per page. Copied 4 Jumādā II, 1082H/October 8, 1671 in Damascus. The title is given here merely as *Kitāb sharḥ al-Talkhīṣ* ("Book of Commentary on the *Talkhīṣ*"). In this manuscript Ibn al-Bannā's passages are preceded by the letter "م" (*mīm*), which stands for *matn* or *mu'allaf*, both of which mean "[original] text" and al-Hawārī's comments start with a "ش" (*shīn*) for *sharḥ* (commentary).

Tehran, Library of Parliament MS 2672/2 ff. 10a-56b, copied before 972H/1564. The title page shows *Kitāb al-lubāb fī sharḥ Talkhīṣ a'māl al-ḥisāb* ("The Essence of Commentary on Condensed [Book] on the Operations of Arithmetic").

Other known manuscripts, which we could not access, are:  
 Madrid, Escorial MS 948/2, incomplete, copied 867H/1462  
 Madrid, Escorial MS 953/1, ff. 2b-79a, copied before 957H/1550  
 London, India Office MS 770/3 ff. 19b-69b copied 856H/1452  
 Rich (Morocco), Ḥamzāwiyya Library MS 145/2, 117 ff  
 Tamegroute (Morocco), al-Khizāna al-Nāsiriya MS 3080  
 Rabat, Bibliothèque Générale MS Q846, 69 ff.  
 Rabat, Bibliothèque al-Ḥasaniyya MS 2186/2, 97 ff, incomplete and mixed up  
 Damascus, al-Zāhiriya Library MS 6666/1, ff. 1b-112b, copied 1002H/1594

Cairo, Central Library of Islamic Manuscripts of the ministry of *waqf*, MS 1077, 80 ff. Copied 1270H/1853

Two manuscripts are reported to contain al-Hawārī's commentary but do not: Istanbul, SM Laleli 2780 and Cairo, Falak 6829/1. ff. 1a-53b, copied 1050H. The second of these is probably Ibn Majdī's commentary. The new digital catalogue of the Cairo Library confirms that MS 6829 is Hawārī's *sharḥ*.

### Comparing al-Hawārī's *al-Lubāb* with his masters' *Talkhīṣ* and *Rafʿ al-ḥijāb*

In *Rafʿ al-ḥijāb* Ibn al-Bannā' expounded on his *Talkhīṣ* with further explanations, linguistic justifications, as well as some numerical examples. By contrast, al-Hawārī wrote in his introduction that his main goal was to provide numerical examples lacking in both the *Talkhīṣ* and *Rafʿ al-ḥijāb*. In fact, the book contains very few elaborations on Ibn al-Bannā's rules. Al-Hawārī reproduces the entire text of the *Talkhīṣ* and places his numerical examples after each of Ibn al-Bannā's explanations.

We have found that 32 of the 284 numerical examples in al-Hawārī's commentary are taken from *Rafʿ al-ḥijāb*. The following table gives the distribution by type of the number of these borrowed examples. Ibn al-Bannā's book is not mentioned in any of them:

|                           | Whole numbers | Fractions | Roots | Scales | Algebra | Total |
|---------------------------|---------------|-----------|-------|--------|---------|-------|
| al-Hawārī's own           | 96            | 32        | 54    | 6      | 64      | 252   |
| from <i>Rafʿ al-ḥijāb</i> | 1             | 13        | 10    | 3      | 5       | 32    |

In addition, eight problems on summing numbers from *al-Lubāb* are also in Ibn al-Yāsamīn's (d. 1204) *Talqīḥ al-afkār fī'l-ʿilm bi-rushūm al-ghubār* ("Grafting of opinions of the work on dust-figures").<sup>13</sup> In an unrelated passage al-Hawārī quotes Ibn al-Yāsamīn, so of course he was familiar with that mathematician's work. Most of these summation problems are standard examples that are found in every book that explains the rules, such as adding the consecutive numbers from one to ten, adding the consecutive cubes of one to ten, etc.

The numerical examples in al-Hawārī that are not in *Rafʿ al-ḥijāb* are also not in any other known source, including Ibn al-Bannā's *al-*

<sup>13</sup> [Ibn al-Yāsamīn 1993, 136-144].

*Maqālāt al-arbaʿ*,<sup>14</sup> his algebra book *al-Uṣūl wa'l-muqaddimāt fī'l-jabr wa'l-muqābala* (henceforth *Algebra*),<sup>15</sup> or in the arithmetic books by al-Ḥaṣṣār, Ibn al-Yasamīn, or Ibn Mun'im.<sup>16</sup> It thus appears that these examples are al-Hawārī's own. The vast majority of al-Hawārī's problems are purely arithmetical. Very few of them are framed in the *muʿāmalāt* (business) style.

Al-Hawārī includes seven passages from *Rafʿ al-ḥijāb* that are indicated as coming from the *Talkhīṣ*, but which are not in Souissi's edition.<sup>17</sup> It seems that in the course of dictating his *Talkhīṣ* Ibn al-Bannā' inserted material from his commentary. In three other places al-Hawārī mentions explanations given by Ibn al-Bannā' in which he cites *Rafʿ al-ḥijāb* explicitly.<sup>18</sup>

One passage in al-Hawārī's book belongs to the *Talkhīṣ* but is absent in Souissi's edition. In the section on summing number we find "If the disparity of the numbers is a known number other than doubling, then multiply the disparity by the number of numbers less one. Adding the first number to the result gives the last number. Add it to the first, and multiply it by half of the number of numbers. It yields the required answer."<sup>19</sup> This passage is also attributed to the *Talkhīṣ*

<sup>14</sup> This book was edited by A. S. Saidan with the title *Māqālāt fī'l-ḥisāb* [Ibn al-Bannā' 1984].

<sup>15</sup> Edited under the title *Kitāb al-jabr wa'l-muqābala*, again by Saidan [Saidan 1986, vol 2], and translated into French by Djebbar in his unpublished doctoral thesis: "Mathématiques et Mathématiciens dans le Maghreb médiéval (IXe - XVIe s.)", volume II. Université de Nantes, 1990.

<sup>16</sup> Specifically, al-Ḥaṣṣār's *Kitāb al-bayān wa'l-tadkhār fī ṣanʿat ʿamal al-ghubār* (latter 12th c.), Ibn al-Yasamīn's *Talqīh al-afkār* mentioned above, and Ibn Mun'im's *Fiqh al-ḥisāb* (12th-13th c.).

<sup>17</sup> Paragraphs taken from *Rafʿ al-ḥijāb* but absent from *Talkhīṣ*: how to add successive numbers starting with a number greater than 1 (p. 228.13-16 in [Ibn al-Bannā' 1994]), repeated subtractions (p. 245.8-15), two definitions of division (p. 263.4-9), other lesser known methods of denomination (p. 267.11-14), subtraction of fractions with repeated "and less" (also in *Maqālāt*) (p. 275.8-9), two kinds of fraction conversions (p. 279.18-280.20), and rational vs. surd roots (p. 283.3-7).

<sup>18</sup> Madina MS, ff. 21b.-6, 50a.-3, 51a.-2.

<sup>19</sup> Madina MS fol. 8b.-3. This should be placed just after [Ibn al-Bannā' 1969, 42.12]. The disparity is the difference between consecutive terms. In the example given by al-Hawārī six numbers, starting with 10, have a disparity of three. So the sequence is 10, 13, 16, 19, 22, 25. So 3 (the disparity) multiplied by 5 (the number of number less one) gives 15,

in al-Qalaṣādī's commentary.<sup>20</sup> This section on addition of numbers, including the added paragraph, is nearly identical word-for-word to the treatment in Ibn al-Yāsamīn's *Talqīh al-afkār*. Ibn al-Bannā' seems to have copied it from this book.<sup>21</sup>

A little before this al-Hawārī ascribes the following passage to the *Talkhīṣ*: "If the situation is different, then multiply the remainder by the first <square> to get the required number. A different situation is when the first square is not one." The first sentence is in Ibn al-Yāsamīn's *Talqīh al-afkār*, Souissi's edition of the *Talkhīṣ*, and al-Qalaṣādī's commentary, but the second sentence is not in any of them.

Still other passages in *al-Lubāb* attributed to the *Talkhīṣ* are not in Souissi's edition, or in any other known text, including the commentaries by al-'Uqbānī and al-Qalaṣādī. Here are the translations, with the location in Souissi's edition of the *Talkhīṣ* where they would have gone:

"And by 'power of ten' (*'aqd*) we mean that the first <non-zero> digit is equal to a ten or a hundred or the like." *Place after [Souissi 1969, 50.10].*<sup>22</sup>

"When looking for roots fractions come in four types. In one of them the numerator has a rational root and the denominator does too. Work it out as explained above. In the second type neither of them has a rational root, so work it out by the first rule. In the third type the denominator has a rational root but the numerator does not have a rational root. For this type one can work it out by the first rule or by the second. In the fourth type the numerator has a rational root and the denominator does not have a rational root, so work it out by the first rule". *Place after [Souissi 1969, 64.18]. The rules, with al-Hawārī's numerical examples (omitted here), are also covered in [Ibn Ghāzī 1983, 155ff], but with no direct quotations.*

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which added to 10 (the first number) gives 25 (the last number). The rule for the sum is evident in the example, too.

<sup>20</sup> [al-Qalaṣādī 1999, 52].

<sup>21</sup> [Ibn al-Yāsamīn 1993, 136.6-12]. See [Djebbar 2002, 221].

<sup>22</sup> This is paraphrased in [Ibn Ghāzī 1983, 90.14-15].