# Suhayl

International Journal for the History of the Exact and Natural Sciences in Islamic Civilisation



Volume T





In Cooperation with the Commission on History of Science and Technology in Islamic Societies

## Contents

List of Contributors	7
GLEN VAN BRUMMELEN, TARO MIMURA AND YOUSUF KERAI, Al-Samaw'al's Curious Approach to Trigonometry	9
JOSEP CASULLERAS AND JAN P. HOGENDIJK, Progressions, Rays and Houses in Medieval Islamic Astrology: A Mathematical Classification	33
DAVID A. KING, Two Newly-discovered Astrolabes from 'Abbasid Baghdad	103
MONTSE DÍAZ-FAJARDO, Ibn al-Raqqām's Notes on Practical Geometry	117
DIRK GRUPE, The 'Thābit-Version' of Ptolemy's <i>Almagest</i> in MS Dresden Db.87	147
MIQUEL FORCADA, Innovation and Tradition in Premodern Procedures for Neonatal Care	155
JOSÉ CHABÁS, The Toledan Tables in Castilian: Excerpts of the Planetary Equations	179
JOSÉ BELLVER AND JULIO SAMSÓ, Ibn al-Raqqām on Lunar Parallax	189

#### Contents

# Reviews

CHARLES BURNETT, <i>Numerals and Arithmetic in the Middle Ages</i> (R. Comes).	231
DAVID JUSTE, Les manuscrits astrologiques latins conservés à la Bayerische Staatsbibliothek de Munich (J. Samsó).	246
DAVID A. KING, Astrolabes from Medieval Europe, (R. Puig).	247
AHMED DJEBBAR AND MARC MOYON, Les sciences arabes en Afrique. Mathématiques et astronomie IXe – XIXe siècles. Suivi de la Nubdha fī 'ilm al-ḥisāb d'Aḥmad Bābir al-Arawānī (J. Samsó).	250
SONJA BRENTJES, Travellers from Europe in the Ottoman and Safavid Empires, 16th-17th centuries. Seeking, Transforming, Discarding Knowledge (M. Rius).	253

# Progressions, Rays and Houses in Medieval Islamic Astrology: A Mathematical Classification<sup>1</sup>

Josep Casulleras and Jan P. Hogendijk

**Keywords**: Applied mathematics, medieval astrology, astrology in medieval Islam, planetary aspects, projection of rays, astrological houses, progressions, *tasyīr*, hour lines, position circles, medieval astronomical instruments.

#### Abstract

Medieval Islamic mathematicians and astronomers developed a variety of mathematical definitions and computations of the three astrological concepts of houses, rays (or aspects) and progressions. The medieval systems for the astrological houses have been classified by J.D. North and E.S. Kennedy, and the purpose of our paper is to attempt a similar classification for rays and progressions, on the basis of medieval Islamic astronomical handbooks and instruments. It turns out that there were at least six different systems for progressions, and no less than nine different systems for rays. We will investigate the historical relationships between these systems and we will also discuss the authors to whom the systems are attributed in the medieval Islamic sources.

<sup>&</sup>lt;sup>1</sup> This paper has been finished in the framework of a research programme on "The evolution of Science in al-Andalus' Society from the Early Middle Ages to the Pre-Renaissance and its impact on the European and Arabic cultures (10<sup>th</sup>-15<sup>th</sup> centuries) II", sponsored by the Spanish Ministry of Science and Innovation (FFI2011-30092-C02-01).

### Table of Contents

0.	Preface	.35
1.	Introduction	.36
2.	Basic definitions	.37
	2.1 Houses	.38
	2.2 Rays	
	2.3 Progressions	
3.	Systems for Progressions	.43
	3.1 Progressions for predicting the moment of death	.45
	3.1.0 Direct Method	
	3.1.1 Right Ascension Method	
	3.1.2 Oblique Ascension Method	
	3.1.3 Position Semicircle Method	
	3.1.4 Hour Line Method	
	3.1.5 Distance Method	
	3.1.6 Hypothetical use of the Prime Vertical	
	3.2 Progressions for predictions for a given date	
4.	Systems for Casting the Rays	
	4.1 Ecliptic Methods	
	4.2 Right Ascension Method	
	4.3 Oblique Ascension Method	
	4.4 Single Position Semicircle Method	
	4.5 Single Hour Line Method	
	4.6 Four Position Circles Method	
	4.7 Standard Houses Method	
	4.8 Seven Hour Lines Method	.75
	4.9 Use of the Prime Vertical	
	4.10 Mathematical Properties of the Methods for Casting the Rays	
5.	Houses	
	5.1 Equatorial (Fixed Boundaries) Method	.80
	5.2 Prime Vertical Method	.82
	5.3 Hour Lines Method	
	5.3.1 Split Differences Method	
	5.4 Standard Method	
6.	Final remarks	
	6.1 Ptolemy and Hermes	
	6.2 Chronological survey of systems and their relations	
	6.2.1 A.D. 0 - 200	

6.2.2 Late antiquity	90
6.2.3 Sassanid Iran	
6.2.4 Ninth-century Baghdād	90
6.2.5 The tenth and eleventh centuries	
6.2.6 From the eleventh century on	92
6.3 Open questions	
7. References	

#### 0. Preface

In November 1998, J.P. Hogendijk presented at a conference in the Dibner Institute a preliminary classification of medieval Islamic methods for defining the two astrological concepts of "progressions" and "projection of rays" (or "planetary aspects"). These concepts are associated with natal astrology but unconnected to the medieval practice of astronomy, just like the (related) division of the ecliptic into twelve astrological houses. Hogendijk's preliminary survey was inspired by the classification of the astrological houses by J.D. North (1986) and E.S. Kennedy (1996). The survey was not published at that time, but a photocopy of the lecture (Hogendijk 1998) was widely circulated among historians of Islamic science.

In October 2006, J. Casulleras published as his doctoral thesis an edition with translation and commentary of the entire Treatise on the Projection of Rays by the eleventh-century Andalusian mathematician and astronomer Ibn Mu'ādh al-Jayyānī (died 1093). Only parts of this treatise had been studied before (Kennedy 1994, Casulleras 2004 and Hogendijk 2005). In the course of his research, Casulleras investigated numerous relevant sources on the three above-mentioned astrological concepts (cf. Casulleras 2010, pp. 49-170) which had not been covered in Hogendijk's preliminary classification. Many of these sources belong to the Western Islamic astrological tradition. It therefore seemed natural to us to produce a more definitive version of the previous classification as a joint work. This is what we intend to offer in the following pages. Although numerous changes and additions have been made to Hogendijk's lecture of 1998, and sub-sections have been added, we have maintained as much as possible the previous numbering of sections in order to be consistent with the citations in the recent research literature to the

photocopied version of the lecture.<sup>2</sup> Only in one case we have changed the original structure: Section 4.8 in the 1998 lecture (Mathematical Properties of the Equatorial Methods) corresponds to our current Section 4.10.

#### 1. Introduction

This paper concerns the history of applied mathematics in medieval Islamic civilization.<sup>3</sup> We will discuss the three astrological concepts of houses, rays and progressions and their mathematical interpretations. In Section 2 we introduce these concepts and explain their importance for the astrologers. The houses, rays and progressions could be defined according to mathematically different systems. The medieval systems for defining the houses were first classified by J.D. North (1986), and his classification was extended by E.S. Kennedy (1996). The purpose of this paper is to attempt a classification for rays and progressions similar to the classification which North and Kennedy gave for the houses. Our classification is based on the publications of many modern historians of Islamic science. A definition and reference for the use of progressions among the Arabs was given by O. Schirmer (1934), whereas a significant study on the rays in the Islamic area was published by E.S. Kennedy and H. Krikorian-Preisler (1972). An overview of the resolution of astrological questions by means of analogical instruments in al-Andalus was presented in 1990 by Calvo (published in 1998). Relevant medieval Arabic sources have been published or analyzed,<sup>4</sup> important older publications on Islamic astronomical instruments have been collected in Sezgin 1990-1991. and a comprehensive monograph on the same subject has been published by King (2005). It turns out that there were at least six different systems for

<sup>&</sup>lt;sup>2</sup> Some of the publications that have used Hogendijk's original work, in chronological order, are: Samsó 1999, p. 117 (in the original) =19 (in the reprint); Samsó and Berrani 1999, pp. 302, 306, 310; Samsó 2004, p. 510; Samsó and Berrani 2005, pp. 188, 199, 202, 203, 204, 220; Casulleras 2007a, passim; Casulleras 2007b, pp. 48, 50, 52-55, 57-59, 61-63, 77; Díaz Fajardo 2008, pp. 24, 29, 30, 31; Casulleras 2008/2009, passim; Samsó 2009, pp. 31, 33; Casulleras 2009, passim; Díaz Fajardo 2010, pp. 302, 304, 305, 307, 311, 320, 322; Casulleras 2010, passim; Díaz Fajardo 2011, pp. 336, 340.

<sup>&</sup>lt;sup>3</sup> The adjective "Islamic" in a term like "Islamic astrology" refers to culture and not to religion. Astrology was strongly condemned by some medieval Islamic theologians because they felt that it contradicted the omnipotence of God.

<sup>&</sup>lt;sup>4</sup> We have not compiled a list of all the sources and studies that might be connected to the subject of the computations in medieval Arabic astrology, but an outline of it can be extracted from the references given at the end of this paper.

defining progressions, to be discussed in Section 3 of this paper, and no less than nine different systems for defining the rays, to be discussed in Section 4. We consider two systems as different if they have different (that is, mathematically inequivalent) geometric definitions. One geometric system can lead to different numerical computations.

For many medieval Islamic astrologers, the rays and progressions were not independent from each other and from the houses, and thus the history of one of these concepts may shed light on the history of the others. In Section 5 we introduce the systems of houses from the classification of Kennedy and North which are related to systems for progressions and rays. In Section 6 we discuss the usual attributions of the methods, we summarize some of their historical relationships, and we pose some questions deserving further research.

Sections 3 and 4 of our paper also involve Greek astrology, in particular the mathematical aspects of the *Tetrabiblos* of Ptolemy (ca. A.D. 150). We have added some references to medieval Hebrew and Latin texts but we have not really investigated the transmission and further development of progressions and rays in the medieval Hebrew and Latin scientific traditions. Our survey of the Islamic systems for progressions and rays is incomplete because there are many unpublished sources which we have not been able to consult. Nevertheless, our paper gives a general idea of the nature and the scope of these non-trivial applications of mathematics in medieval Islamic civilization. These applications are not only historically important, but to our mind they are also fascinating because they are so strange to modern scientific eyes. Future historians may well regard some present-day applications of mathematics with similar feelings of fascination.

#### 2. Basic definitions

This section contains simple explanations of the astrological *houses*, *rays* and *progressions*. We assume that the reader has a rudimentary knowledge of spherical geometry and astronomical coordinate systems on the celestial sphere, as explained in any introduction to spherical astronomy (for example, Smart 1939).

#### 2.1 Houses

The ancient Greek astrologers introduced the division of the ecliptic into twelve "houses". This division depends on the horizon, and it is therefore not the same as the division of the ecliptic into the zodiacal signs Aries, Taurus, etc. In this paper we will only be concerned with systems based on the four cardinal points, that is to say the intersections of the ecliptic with the horizon and meridian planes. In these systems the beginnings of the first, fourth, seventh and tenth houses are defined as the ascendent or rising point on the Eastern horizon (1), the intersection between ecliptic and the meridian below the horizon (4), the descendant or setting point on the Western horizon (7), and the midheaven, that is the intersection between ecliptic and the meridian above the horizon (10) (Figure 1).

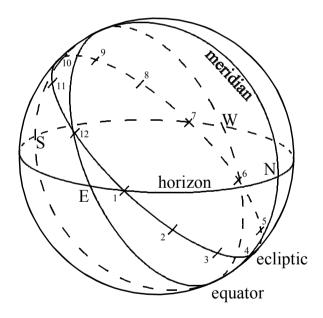


Figure 1

<sup>&</sup>lt;sup>5</sup> See Bouché-Leclercq 1899, pp. 279.

<sup>&</sup>lt;sup>6</sup> We will not be concerned here with systems where the beginning of the first house was 5 degrees away from the rising point. See for houses in general North 1986.

There were many different systems for the definition of the second, third, fifth, sixth, eighth, ninth, eleventh and twelfth houses in medieval Islamic astrology. In the simplest system, which was already used in ancient horoscopes (North 1986, p. 6, fn. 13), the remaining boundaries between the houses were obtained by trisecting each of the four arcs between two cardinal points. The Iranian astronomer al-Bīrūnī<sup>7</sup> (973-1046) called this system the "method of the ancients" (al-Bīrūnī 1954-1956, vol. 3, p. 1356; North 1986, p. 40), and it is called the "Dual Longitude Method" in the North-Kennedy classification.

In all systems, the first house is the part of the sphere below the Eastern horizon, and the houses are numbered counter-clockwise, for an observer in the temperate regions on the Northern hemisphere who looks to the south. The process of defining the houses is called in Arabic the "equalization of the houses", and the beginning points are called the cusps of the houses in the modern literature, and the centres of the houses in Arabic.

During one daily revolution of the celestial sphere, <sup>11</sup> any celestial body will pass through all twelve houses. These houses were connected to many things, including life (1), possessions (2), brothers and sisters (3), parents (4), children (5), illness (6), marriage (7), death (8), travels (9), honours (10), friends (11), and enemies (12). <sup>12</sup> Lengthy descriptions of the qualities of the houses can be found in the work of the early tenth-century Christian astrologer Ibn Hibintā (1987, vol. 1, pp. 52-124). The qualities of the first, second, etc. houses were often connected to the qualities of the first, second, etc. signs of the zodiac.

Many modern astrologers define the houses according to the so-called systems of Regiomontanus, Campanus and Placidus. These three systems, all of medieval Islamic origin, will be mentioned in Sections 5.1-5.3.

<sup>&</sup>lt;sup>7</sup> On al-Bīrūnī see, for example, Yano 2007 and the references given there.

<sup>&</sup>lt;sup>8</sup> taswiyat al-buyūt.

<sup>&</sup>lt;sup>9</sup> We do not know the origin of this term.

<sup>&</sup>lt;sup>10</sup> markaz, pl. marākiz.

<sup>&</sup>lt;sup>11</sup> This apparent revolution for a modern reader, who knows that the earth revolves around its axis, was a real revolution for a medieval astronomer.

<sup>&</sup>lt;sup>12</sup> See Bouché-Leclercq 1899, pp. 280-285.

#### 2.2 Rays

The ancient Greek astrologers also introduced the doctrine that each planet  $^{13}$  P casts seven visual rays to other points  $P_1 \dots P_7$  of the ecliptic. The positions of these rays are defined by means of a regular hexagon, a square and an equilateral triangle.  $^{14}$  In the simplest system, these polygons are inscribed in the ecliptic with an angular point at P, as in Figure 2, in which the arrow indicates the direction of increasing celestial longitude. The "rays"  $PP_1$ ,  $PP_2$ ,  $PP_3$  ( $PP_7$ ,  $PP_6$ ,  $PP_5$ ) are called the left (right) sextile, quartile and trine rays respectively, and  $PP_4$  is the ray to the diametrically opposite point. If another celestial body Q happens to be close to  $P_b$  Q is said to be "looked at" (Latin: adspectus) and the two bodies P and Q are said to make an aspect. This information is astrologically significant. For the interpretation, the theory of rays and aspects between the planets was often related to the theory of the aspects between the zodiacal signs. The trine ray or aspect was regarded as beneficial, the sextile ray or aspect as less beneficial, the quartile ray or aspect as damaging, while the opinions diverged on the qualities of the opposite ray and aspect.

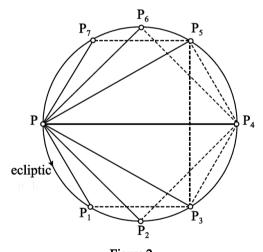


Figure 2

<sup>&</sup>lt;sup>13</sup> In this connection, a planet is a celestial body which changes its position relative to the fixed stars. Thus the sun and moon are also considered to be planets.

<sup>&</sup>lt;sup>14</sup> See Bouché-Leclercq 1899, pp. 177-178, 247-251 for the theory in antiquity.

<sup>&</sup>lt;sup>15</sup> For this theory see Bouché-Leclercq 1899, pp. 165-177.

The medieval Islamic astrologers enthusiastically adopted the theory of the seven rays. Al-Bīrūnī points out that the planets cast light in all directions, not only in the direction of the astrological rays, but he argues that the points  $P_i$  have some special influence, and that this effect can be compared to the theory of musical intervals. He also notes the connection between the tides and the quartile aspects between the moon and the sun (al-Bīrūnī 1954-1956, pp. 1379-1382).

The above-mentioned system for defining the rays (Figure 2) is still in use by modern astrologers, but it was not the most popular system in medieval Islamic astrology. In Section 4 we will list some examples of its use and we will discuss the more complicated systems which were preferred by most medieval Islamic astrologers, and which still await rediscovery by their modern successors.

#### 2.3 Progressions

The ancient Greek astrologers also introduced the doctrine that various events in the life of an individual could be predicted on the basis of the positions of the celestial bodies at the moment of birth. This doctrine was accepted and elaborated in Islamic astrology. We will explain the general idea in connection with the prediction of the moment of death of the individual (Figure 3). First the astrologer had to select two celestial bodies or other significant points (for example the ascendent, or one of the seven rays of a planet) in the celestial configuration at the moment of birth of the individual. The way in which these bodies or points were selected depended on astrological arguments which do not concern us here, although they were of course decisive for the resulting predictions. <sup>16</sup> One of these points was thought as emitting life-force, the other as a destructive point which destroys life. <sup>17</sup>

We now fix the emitting point P in its initial position and we rotate the destructive point F around the celestial axis (i.e. the line through the celestial North Pole C and the centre of the sphere) until it reaches P. If this happens

<sup>17</sup> The first point was called in Greek the *aphetic* body (Bouché-Leclercq 1899, pp. 415-416), in Arabic the *haylāj*, from Persian *hīlāj*, "lord of a building" (Nallino 1907, vol. 2, p. 355) and in Latin the *hyleg* or *significator*. The other body was called the *anairetic* body in Greek, *qāṭi* in Arabic, and *promissor* in Latin (Schirmer 1934).

<sup>&</sup>lt;sup>16</sup> See Bouché-Leclercq 1899, pp. 411-422.

after rotation over n degrees, <sup>18</sup> the individual will live n solar years. Similarly, a rotation over one minute of arc corresponds to approximately six days of the life of the individual. The destructive point F has now "progressed" from its initial position, and the method is called "progression" or in Arabic  $tasy\bar{n}$ . The emitting and the destructive points were always selected in such a way that the great circle distance between them is at most 90 degrees, so that n < 90.

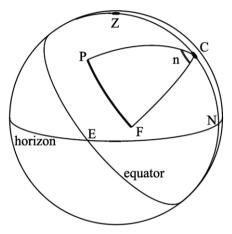


Figure 3

In Figure 3, the emitting point P "precedes" F in the sense that P rises earlier than F on the Eastern horizon, and the rotation of F to P is in the direction of the daily motion of the celestial sphere. This situation was the most common one, and it will be assumed in the rest of this paper. Some astrologers were also willing to consider the destructive point "preceding" the emitting point, and rotation in the direction opposite to the daily motion of the sphere (see e.g. Ptolemy 1940, pp. 279-281, Ibn Hibintā 1987, vol. 1, p. 139).

<sup>&</sup>lt;sup>18</sup> The ancient and medieval astrologers expressed this quantity not as an angle but as a rotation arc on the celestial equator.

<sup>&</sup>lt;sup>19</sup> This technical term, literally: "making travel", is often used in modern articles on the history of the subject. Point *F* is also called *al-musayyar*, the point which is made to travel, and point *P al-musayyar ilay-hi*, the point toward which the travel is made.

<sup>&</sup>lt;sup>20</sup> This terminology was used by al-Battānī (Nallino 1899, vol. 3, pp. 198-203) and al-Bīrūnī (1954-1956, p. 1393).

Figure 3 is drawn for the case where points P and F have the same declination, so the rotated point F can coincide with the initial position of P, and vice versa. Some astrologers believed that the progression could only have an effect in this case. The eleventh-century Maghribī astrologer Ibn Abī-l-Rijāl copies from Ibn Hibintā an approximate solution for checking whether the equatorial declinations of the two elements implied are the same, and mentions Dorotheus of Sidon<sup>21</sup> as his source<sup>22</sup> (Ibn Abī-l-Rijāl 1954, pp. 175-176; Ibn Hibintā 1987, vol. 1, p. 144; Díaz Fajardo 2008, pp. 95-103; Casulleras 2010, pp. 108-111). However, in almost all cases points P and F do not have the same declination, so the rotated point F can never coincide with P. In the next section we will see how the Greek and Islamic astrologers generalized the method for this case also.

Modern astrologers still predict the events in the life of an individual from the constellation at the moment of birth by letting the celestial bodies "progress" from their original positions, but they do not use the (apparent) daily rotation of the celestial sphere in this connection. Instead, they simply obtain the arc of the  $tasy\bar{t}r$  on the ecliptic, a method which, as we shall see, is only one of a variety of alternatives existing in medieval sources.

In our presentation thus far, the concepts of houses, rays and progressions seem to be mathematically unrelated. In the next sections we will see that some systems for the mathematical definition of progressions were also adapted to define rays and houses, and that another system for houses was adapted to the rays. In another case of cross-fertilization, we have not been able to find out whether the system was first applied to the houses or to the rays. We will discuss the individual systems in Sections 3-5 and we will list the relationships in a chronological survey in Section 6.2.

#### 3. Systems for Progressions

In this and the following sections we use the following notation for any celestial body P or any point P on the (moving) celestial sphere:

•  $\alpha_{\varphi}(P)$  is the oblique ascension of P in a locality with Northern geographical latitude  $\varphi$ . This is the length of the arc of the celestial equator which

<sup>&</sup>lt;sup>21</sup> Dorotheus of Sidon was a Greek astrologer who lived in the first century A.D. On the Arabic traces of his works see (Sezgin 1974-1984, vol. 7, pp. 32-38; Pingree 1978).

<sup>&</sup>lt;sup>22</sup> Díaz Fajardo (2008, p. 98) notes that the quotation is not found in the preserved works of Dorotheus (1976).

rises on the Eastern horizon in the time interval which begins with the rising of the vernal point V and ends with the next rising of P(Figure 4).

- $\alpha_0(P)$  is the right ascension of P.
- $\alpha_{-\varphi}(P)$  is the oblique descension of P, i.e. the length of the arc of the celestial equator which sets on the Western horizon in the time interval which begins with the setting of the vernal point and ends with the next setting of P, at a locality with Northern geographical latitude  $\varphi$ . This quantity is equal to the oblique ascension in a locality with southern latitude  $-\varphi$ ; such latitudes were sometimes considered by the Islamic astronomers.

Oblique ascensions and descensions are usually defined only for points P on the ecliptic (see Figure 4, in which  $\overline{\varphi} = 90^{\circ} - \varphi$ ,  $\varepsilon \approx 23^{\circ}$  30' the obliquity of the ecliptic, and Pedersen 1974, pp. 99-113, for all details, including computation and tables). We use this notation for arbitrary points P for sake of brevity.

All angles and arcs will be reckoned modulo 360. Thus,  $40^{\circ} - 320^{\circ} = 80^{\circ}$ .

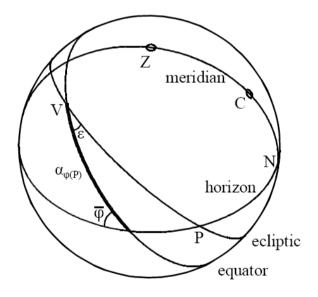


Figure 4

<sup>&</sup>lt;sup>23</sup> Cf. Hogendijk 1989, pp. 176-178.